

CSM—65/22

STATISTICS

PAPER—II

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Time : 3 Hours

Full Marks : 250

*The figures in the right-hand margin indicate marks.
Candidates should attempt **any 10 (ten)** questions of
GROUP—A with word limit of 250 words and should
attempt **any 5 (five)** questions from **GROUP—B**
with word limit of 300 words.*

GROUP—A

Attempt **any 10 (ten)** questions from the following :

1. (a) Explain what you mean by 'Statistical Quality Control'.
Distinguish between 'Product Control' and 'Process Control'. 10
(b) Why \bar{X} and R control charts should be used simultaneously? 5
2. (a) Describe how a control chart for fraction defective is set. What
modification is needed, if varying numbers are inspected on
different occasions? 10
(b) Discuss the role of C-chart in Statistical Quality Control. 5
3. (a) Distinguish between Producer's risk and Consumer's risk. 6
(b) Describe single sampling inspection plan for attributes and obtain
OC curve for this plan, clearly stating the assumptions used. 9
4. (a) Consider a random variable T with its lifetime distribution having
probability density function :
$$f(t) = \lambda \alpha t^{\alpha-1} \exp(-\lambda t^\alpha); t \geq 0, \lambda > 0, \alpha > 0$$

Find the hazard rate function ($h(t)$) of T and interpret it. 7
(b) Consider a system consisting of n components such that the
failure of the i^{th} components occurs in accordance with a Poisson
Process of intensity a_i . Find the reliability of the system under :
(i) Series system
(ii) Parallel system 4+4=8

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5. (a) The two independent components joined in parallel have hazard rates $h_1(t)=1$ and $h_2(t)=2$, $t \geq 0$. Comment on the ageing properties of the system. 8

(b) Give a graphical representation/rough sketch of Bathtub hazard function curve (without using any graph sheet) and interpret it statistically. Suggest any two practical situations, where you would expect a Bathtub type of hazard function curve. 7

6. (a) Derive the Fisher information based on one observation with Type I censoring, for an exponential distribution. 7

(b) Derive the Fisher information based on n units put on test and experimenter decides to call off the experiment as soon as m failures have been observed, for an exponential distribution. 8

7. (a) Discuss the uses and limitations of index numbers. 7

(b) Discuss the uses and construction of Consumer Price Index. 8

8. (a) What is time series? State its utility in economics and business world. 7

(b) Find the first three autocorrelation coefficients of the following moving average process :

$$X_t = 1.6e_t - 0.8e_{t-1} + 0.4e_{t-2} - 0.2e_{t-3}$$

Here $\{e_t\}$ is a purely random process with mean zero and variance σ^2 . 8

9. (a) Let Y_1, Y_2, Y_3, Y_4 are independent with

$$E(Y_1) = E(Y_2) = \theta_1 + \theta_2$$

$$E(Y_3) = E(Y_4) = \theta_1 + \theta_3$$

$$\text{Var}(Y_i) = \sigma^2, i = 1, 2, 3, 4$$

Determine the condition of estimability of linear parameter function $\underline{l}'\underline{\theta} = l_1\theta_1 + l_2\theta_2 + l_3\theta_3$. 7

(b) Show that the best estimator of the estimable linear parametric function $\underline{l}'\underline{\theta} = l_1\theta_1 + l_2\theta_2 + l_3\theta_3$ is $\underline{l}'\hat{\underline{\theta}} = l_1\hat{\theta}_1 + l_2\hat{\theta}_2 + l_3\hat{\theta}_3$, where $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3$ are a set of least square estimators of $\theta_1, \theta_2, \theta_3$ clearly stating the assumption used if any. 8

10. Explain the role of official statistical system in India for collecting data relating to :

(a) Crop production

(b) Industrial production

7+8=15

11. Discuss the important function performed by the following Statistical agencies in India :

(a) Central Statistical Organisation

(b) National Sample Survey Organisation

8+7=15

12. (a) Write down the important recommendations of National Statistical Commission set up by the Government of India. 9

(b) What steps (in general) the Central and State Governments have taken to implement the recommendations of National Statistical Commission? 6

GROUP—B

Attempt any 5 (Five) questions from the following :

13. (a) Using (i) graphical method and (ii) M-technique, solve the following linear programming problem :

Minimize $Z = 12x + 20y$; subject to $3x + 4y \geq 50$, $7x + 12y \geq 120$ and $x, y \geq 0$. 3+9=12

(b) Give the physical interpretation of a saddle point. Find the range of values for p and q that will render the cell (2, 2) as a saddle point in the game with the following pay-off matrix : 3+5=8

$$\begin{array}{c} \text{Player B} \\ \left(\begin{array}{ccc} 1 & q & 6 \\ p & 5 & 10 \\ 6 & 2 & 3 \end{array} \right) \\ \text{Player A} \end{array}$$

14. (a) A manufacturing company has determined from an analysis of its accounting and production data for a certain part that (i) its demand is 9000 units per annum and is uniformly distributed over the year, (ii) its cost price is ₹ 2 per unit, (iii) its ordering cost is ₹ 40 per order and (iv) the inventory carrying charge is 9% of the inventory value. Further it is known that the lead-time is uniform and equals 8 working days and that the total working days in a year are 300.

Determine the economic order quantity and optimum number of orders per year. 12

(b) What is a replacement problem? Describe some important replacement situations and policies. 8

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15. (a) Define a Markov Chain. Consider the following two-state Markov Chain :

$$P = \begin{pmatrix} 1-a & a \\ b & 1-b \end{pmatrix}, 0 < a < b < 1$$

Find $\lim_{n \rightarrow \infty} P^n$.

3+7=10

- (b) Describe $M/G/1$ queuing model and derive the Pollaczek-Khinchine (PK) formula associated with this model. 10

16. (a) In the context of demographic data, explain the difference between a rate and a ratio. 5

- (b) From the data given below, calculate General Fertility Rate (GFR), Age Specific Fertility Rate (ASFR) and Total Fertility Rate (TFR). 15

Age of Women (Years)	Number of Women in the Specified Age Group	Number of Birth to Women of Specified Age Group
15-19	5680	120
20-24	5324	280
25-29	4720	262
30-34	3935	165
35-39	3675	115
40-44	3025	28
45-49	2601	6
Total	28960	976

17. (a) Fill in the blanks marked X in a portion of a life table given below clearly stating the assumptions used, if any. 13

Age	l_x	d_x	q_x	P_x	L_x	T_x	e_x^0
4	95000	500	X	X	X	4850300	X
5	X	400	X	X	X	X	X

- (b) Describe various sources of demographic data in India. 7

18. (a) Define the term 'validity' and discuss the different concepts of validity. 8

- (b) Give a brief outline of factor analysis and discuss its importance in psychometric studies. 12

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