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CSM—46/22

MATHEMATICS

ଗଣିତ

PAPER—I

Time : 3 Hours

ସମୟ : ୩ ଘଣ୍ଟା

Full Marks : 250

ପୂର୍ଣ୍ଣ ସଂଖ୍ୟା : ୨୫୦

The figures in the right-hand margin indicate marks.

ପ୍ରଶ୍ନପତ୍ରର ଡ଼ାହାଣ ପଟେ ପ୍ରତ୍ୟେକ ପ୍ରଶ୍ନର ମାର୍କ ଦର୍ଶାଯାଇଛି ।

Candidates should attempt ***any 10 (ten)*** *questions of* **GROUP—A** *with word limit of 250 words and should attempt* ***any 5 (five)*** *questions from* **GROUP—B** *with word limit of 300 words.*

ପରୀକ୍ଷାର୍ଥୀମାନେ **GROUP—A** ରୁ ଯେକୌଣସି **୧୦**ଟି ପ୍ରଶ୍ନର ଉତ୍ତର ୨୫୦ ଶବ୍ଦ ମଧ୍ୟରେ ଏବଂ **GROUP—B** ରୁ ଯେକୌଣସି **୫**ଟି ପ୍ରଶ୍ନର ଉତ୍ତର ୩୦୦ ଶବ୍ଦ ମଧ୍ୟରେ ସୀମିତ ରଖିବେ ।

GROUP—A

1. Prove that every homomorphic image of a group G is isomorphic to some quotient group of G. 15

ପ୍ରମାଣ କର ଯେ G group ର ପ୍ରତ୍ୟେକ homomorphic image G ର କିଛି quotient group ପାଇଁ isomorphic ଅଟେ ।

2. Prove that if H be a normal subgroup of a group G and K is a normal subgroup of G containing H, then G/K is isomorphic to $(G/H)/(K/H)$. 15

ପ୍ରମାଣ କର ଯେ H ଯଦି ଗୋଟିଏ group G ର ଏକ ସାଧାରଣ subgroup ଏବଂ K ହେଉଛି G ଧାରଣ କରିଥିବା G ର ଏକ ମାଧାରଣ subgroup, ତେବେ G/K ହେଉଛି $(G/H)/(K/H)$ ପ୍ରତି isomorphic ।

3. Let G be a finite group and p be a prime. If p^m divides $o(G)$, then prove that G has at least one subgroup of order p^m. 15

G କୁ ଏକ finite group ହେବାକୁ ଦିଅ ଏବଂ p^m କୁ ଏକ prime ହେବାକୁ ଦିଅ । ଯଦି p^m, $o(G)$ କୁ ବିଭାଜନ କରେ, ତେବେ ପ୍ରମାଣ କର ଯେ G ରେ ଅତି କମ୍‌ରେ ଗୋଟିଏ subgroup order ଅଛି ।

4. Prove that an ideal S of the ring of integers I is maximal if and only if S is generated by some prime integer. 15

ପ୍ରମାଣ କର ଯେ integers I ର ଏକ ideal S ସର୍ବାଧିକ ଅଟେ, ଯଦି ଏବଂ କେବଳ ଯଦି S କିଛି prime integer ଦ୍ୱାରା ଉତ୍ପନ୍ନ ହୁଏ ।

5. In $V_3(R)$, where R is the field of real numbers. Examine each of the following sets of vectors for linearly independent : 15

$V_3(R)$ ରେ, ଯେଉଁଠାରେ R ହେଉଛି ପ୍ରକୃତ ସଂଖ୍ୟା ଗୁଡ଼ିକର କ୍ଷେତ୍ର । Linearly independent ପାଇଁ ନିମ୍ନଲିଖିତ ପ୍ରତ୍ୟେକ ଭେକ୍ଟର ସେଟ୍‌ଗୁଡ଼ିକ ପରୀକ୍ଷା କର :

(i) $\{(2, 1, 2), (8, 4, 8)\}$

(ii) $\{(1, 2, 0), (0, 3, 1), (-1, 0, 1)\}$

(iii) $\{(-1, 2, 1), (3, 0, -1), (-5, 4, 3)\}$

6. Show that the vectors $(1, 2, 1)$, $(2, 1, 0)$, $(1, -1, 2)$ form a basis of $R^3(R)$. 15

$R^3(R)$ ର ଏକ ଆଧାରରୁ ଭେକ୍ଟର $(1, 2, 1)$, $(2, 1, 0)$, $(1, -1, 2)$ କୁ ଦର୍ଶାଅ ।

7. Prove that every n dimensional vector space $V(F)$ is isomorphic to $V_n(F)$. 15

ପ୍ରମାଣ କର ଯେ ପ୍ରତ୍ୟେକ n dimensional vector space $V(F)$ $V_n(F)$ ପ୍ରତି isomorphic ଅଟନ୍ତି ।

8. The mapping $f : V_3(F) \to V_2(F)$ defined by $f(a_1, a_2, a_3) = (a_1, a_2)$ is a homomorphism of $V_3(F)$ onto $V_2(F)$. What is the kernel of this homomorphism? 15

Mapping $f : V_3(F) \to V_2(F)$ defined by $f(a_1, a_2, a_3) = (a_1, a_2)$ $V_3(F)$ onto $V_2(F)$ ଏକ homomorphism ଅଟେ । ଏହି homomorphism ର kernel କ'ଣ?

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9. In what direction, a line be drawn through the point (1, 2), so that its point of intersection with the line $x+y=4$ is at a distance $[(\sqrt{6})/3]$ from the given point? 15

କେଉଁ ଦିଗରେ ବିନ୍ଦୁ (1, 2) ମଧ୍ୟରେ ଏକ ରେଖା ଆଙ୍କିତ ହେବ, ଯାହାଫଳରେ ଦିଆଯାଇ ଥିବା ବିନ୍ଦୁଠାରୁ $[(\sqrt{6})/3]$ ଦୂରତାରେ $x+y=4$ ରେଖା ସହିତ ଏହାର point of intersection ରହିବ ।

10. Find the equation of circle through the points of intersection of $x^2+y^2-1=0$, $x^2+y^2-2x-4y+1=0$ and touching the line $x+2y=0$. 15

$x^2+y^2-1=0$, $x^2+y^2-2x-4y+1=0$ ର points of intersection ଏବଂ $x+2y=0$ touching the line ମାଧ୍ୟମରେ ବୃତ୍ତର ସମୀକରଣ ବାହାର କର ।

11. Show that the equation $y^2+6y-2x+5=0$ represents parabola. Find its vertex, focus, length of latus rectum, equation of axis and directrix. 15

ଦର୍ଶାଅ ଯେ $y^2+6y-2x+5=0$ ସମୀକରଣ parabola କୁ ପ୍ରତିନିଧିତ୍ୱ କରେ । ଏହାର vertex, focus, length of latus rectum, equation of axis ଏବଂ directrix ଖୋଜ ।

12. The points $A(3, 2, 0)$, $B(5, 3, 2)$ and $C(0, 2, 4)$ are the vertices of a triangle. Find the distance of the point A from the point in which bisector of angle BAC meets $[BC]$. 15

$A(3, 2, 0)$, $B(5, 3, 2)$ ଏବଂ $C(0, 2, 4)$ ବିନ୍ଦୁଗୁଡ଼ିକ ଗୋଟିଏ triangle ର vertices ଅଟେ । A ବିନ୍ଦୁର ଦୂରତା ଖୋଜ, ଯେଉଁ ବିନ୍ଦୁରେ BAC କୋଣର bisector $[BC]$ ଠାରେ meet କରେ ।

GROUP—B

13. *(i)* Check the sequence $\{a_n\}$ defined as

$$a_n = 1+\frac{1}{6}+\frac{1}{11}+\cdots\cdots\cdots\cdots+\frac{1}{5n-4}$$

is Cauchy sequence or not.

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ପରିଭାଷିତ କ୍ରମ $\{a_n\}$ ଯାଞ୍ଚ କର

$$a_n = 1 + \frac{1}{6} + \frac{1}{11} + \cdots\cdots\cdots\cdots + \frac{1}{5n-4}$$

Cauchy sequence କି ନୁହେଁ?

(ii) Test for the convergence for the following series :

$$\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \frac{x^4}{4.5} + \cdots\cdots, \text{ where } x > 0.$$ 20

ନିମ୍ନ series ପାଇଁ convergence ନିମିତ୍ତ ପରୀକ୍ଷଣ :

$$\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \frac{x^4}{4.5} + \cdots\cdots, \text{ ଯେଉଁଠାରେ } x > 0.$$

14. *(i)* Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though Cauchy-Riemann equations are satisfied thereof.

Function $f(z) = \sqrt{|xy|}$ origin ରେ analytic ନୁହେଁ ଯଦିଓ Cauchy-Riemann equation ଗୁଡ଼ିକ ଏଥିରେ ସନ୍ତୁଷ୍ଟ, ଦର୍ଶାଅ

(ii) Find the sum of the residues of $f(z) = \frac{\sin z}{z \cos z}$ at its poles inside the circle $|z| = 2$. 20

Circle $|z| = 2$ ଭିତରେ ଥିବା poles ରେ $f(z) = \frac{\sin z}{z \cos z}$ ର sum of the residues କୁ ବାହାର କର ।

15. *(i)* Using the ε-δ definition, prove that $f(x) = \sqrt{x}$ is differential at $x = 3$.

ε-δ ସଂଜ୍ଞା ବ୍ୟବହାର କରି, ପ୍ରମାଣ କର ଯେ $f(x) = \sqrt{x}$, $x = 3$ ରେ ଭିନ୍ନ ଅଟେ ।

(ii) Find the asymptotes of $(x+y)^2(x+y+2) = x+9y-2$. 20

$(x+y)^2(x+y+2) = x+9y-2$ ର asymptotes ଗୁଡ଼ିକ ବାହାର କର ।

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16. *(i)* Find the area enclosed by the curves $x^2 = 8y$ and $y = \frac{64}{x^2+16}$.

$x^2 = 8y$ ଏବଂ $y = \frac{64}{x^2+16}$ ବକ୍ରଗୁଡ଼ିକ ଦ୍ୱାରା ଆବଦ୍ଧ ସ୍ଥାନକୁ ଖୋଜ ।

(ii) Test for the convergence for the improper integral $\int_0^\infty \frac{\sin x}{x} dx$.

20

Improper integral $\int_0^\infty \frac{\sin x}{x} dx$ ପାଇଁ convergence କୁ ପରୀକ୍ଷଣ କର ।

17. State Stokes' theorem and deduce Green's theorem from Stokes' theorem. Verify Stokes' theorem for $\vec{F} = (x^2 + y^2)i - 2xy\,j$ taken around the rectangle bounded by the lines $x = \pm a, y = 0, y = b$. 20

Stokes' theorem କୁ ଦର୍ଶାଅ ଏବଂ Green's theorem କୁ Stokes' theorem ରୁ ବାହାର କର । Stokes' theorem କୁ ଯାଞ୍ଚ କର $\vec{F} = (x^2 + y^2)i - 2xy\,j$, $x = \pm a, y = 0, y = b$ ରେଖାଗୁଡ଼ିକ ଦ୍ୱାରା ସୀମିତ ଆୟତାକାର ଚରିପାଖରେ ନିଆଯାଇଛି ।

18. State Gauss divergence theorem and verify Gauss divergence theorem for $\vec{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ taken over the rectangular parallelepiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$. 20

Gauss divergence theorem କୁ ଦର୍ଶାଅ ଏବଂ Gauss divergence theorem କୁ ଯାଞ୍ଚ କର ।

$0 \le x \le a, 0 \le y \le b, 0 \le z \le c$ ଉପରେ ନିଆଯାଇଥିବା

$\vec{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ ପାଇଁ

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