CSM – 69/21
Statistics
Paper – II

Time : 3 hours

Full Marks : 300

The figures in the right-hand margin indicate marks.

*Candidates should attempt Q. No. 1 from Section – A and Q. No. 5 from Section – B which are compulsory and any **three** of the remaining questions, selecting at least **one** from each Section.*

SECTION – A

1. Answer any **three** questions of the following :

   (a) A random variable, T, has the Weibull distribution with scale parameter λ and shape parameter υ, Find :

   (i) Survival function, S(t), of T.

   (ii) State the intercept and slope of the line if log {– log (S(t))} on the vertical line is plotted against log(t) on the horizontal axis. 10+10 = 20

(b) Discuss the role of Directorate of Economics and Statistics, Government of India. 20

(c) Explain the necessity of analyzing time series data, and describe the various components of times series. 20

(d) Describe the procedure of $\bar{X}$ and S Control Charts when all samples are of same size. How is the procedure modified for variable sample size ? 20

2. (a) 20 samples each of size 10 were inspected. The number of defectives in each of them were : 20

Sample No.	No. of defectives
1	0
2	1
3	0
4	3

Sample No.	No. of defectives
5	9
6	2
7	0
8	7
9	0
10	1
11	1
12	0
13	0
14	3
15	1
16	0
17	0
18	2
19	0
20	0

Construct number defective chart and establish quality standard for future.

(b) Distinguish clearly between fixed base and chain base index number and point out their relative merits and demerits. Calculate fixed base index number from the chain base index number given below : 20

Year	Chain Base Index
1995	98
1996	102
1997	115
1998	135
1999	125
2000	103
2001	95

(c) Explain the problem of multicollinearity in general linear models, and how will you detect it. 20

3. (a) For forecasting the value of a time series, discuss the Box-Jenkins methodology. 20

(b) For a random variable T with the hazard function h(t) as given below, find its survival function and the density function : 20

$$h(t) = \begin{cases} \lambda_1, & t_0 = 0 \le t < t_1 \\ \lambda_2, & t_1 \le t < t_2 \\ \lambda_3, & t_2 \le t < t_3 \\ \vdots & \\ \lambda_{k-1}, & t_{k-2} \le t < t_{k-1} \\ \lambda_k, & t \ge t_{k-1} \end{cases}$$

(c) Describe the procedure and some of the applications of cumulative sum chart (CUSUM chart) for monitoring the process mean. 20

4. (a) Describe the procedure for Sequential sampling inspection plan for attributes and illustrate it with an example. 20

(b) Define Index numbers. Describe the methods of construction of consumer price

index number, and index number of Industrial Production. Also, mention the precautions required while using the consumer price index. 20

(c) Discuss about the principal sources of data relating to organized and unorganized sectors in regard to Statistics of Industrial Production in India. 20

SECTION – B

5. Answer any **three** questions of the following :

(a) Write down the standard form of general linear programming problem clearly stating all the terms involved in it. Also, mention some advantages and limitations of linear programming. 20

(b) Show that Poisson process is a Markov process. 20

(c) Calculate the gross reproduction rate and net reproduction rate from the following data : 20

Age group	Female population ('000)	Female births	Survival rate
15-19	1636	17200	0.872
20-24	1214	68600	0.861
25-29	1482	87500	0.847
30-34	1735	102300	0.832
35-39	1668	81200	0.812
40-44	1612	40100	0.798
45-49	1545	2900	0.743

(d) Four items are to be constructed so that they are equispaced on the difficulty scale. If the easiest item, is passed by 85 percent of the group, the most difficult by 25 percent, find the percentage of individuals in the group passing the other two items. Also, state the assumptions used, if any. 20

6. (a) What are the assumptions of replacement models ? Explain how the theory of replacement is used for replacement of item, which deteriorates with time. The data collected in running a machine, the cost of which is Rs. 60,000 are given below : 18

Year	Cost of spares (Rs.)	Cost of labour (Rs.)	Resale value (Rs.)
1	4000	14000	42000
2	4270	16000	30000
3	4880	18000	20400
4	5700	21000	14400
5	6800	25000	9650

Determine the optimum period for replacement of the machine.

(b) Let $\{X_n\}$, $n = 12, 3, \ldots$ be a Markov chain with state space $\{1, 2\}$ with the following transition probability matrix :

$$\begin{pmatrix} 1 & 0 \\ 0.25 & 0.75 \end{pmatrix}$$

Show that (i) state 1 is recurrent and (ii) state 2 is transient. 6+6 = 12

(c) The mortality rates for a certain population, A, with 1000 births per year are given in the following table where $10q_x$ is the probability that a person aged x years dies within the next 10 years :

Age	$10q_x$
0	0.029
10	0.009
20	0.015
30	0.020
40	0.047
50	0.125
60	0.291
70	0.551
80	0.846
90	0.979
100	1.000

Using only this information, estimate :

(i) The age distribution in 10 year class intervals

(ii) The expected ages at death of groups of people now aged 20 and 40.

20+10 = 30

7. (a) A company distributes its products by trucks loaded at its only loading station. Both, company's trucks and contractor's trucks are used for this purpose. It was found out that on an average every five minutes, one truck arrived and the average loading time was three minutes. 50% of the trucks belong to the contractor. Find :

(i) The probability that a truck has to wait

(ii) The waiting time of truck that waits

(iii) The expected waiting time of contractor's trucks per day, assuming a 24 hours shift. 5+5+10 = 20

(b) For the following pay-off matrix, find the value of the game and the strategies of players A and B by using linear programming : 20

	Player B 1 2 3
Player A 1 2	3 −1 4 6 7 −2

(c) Describe the logistic curve for population growth and state its properties. 20

8. (a) A farmer has 1000 acres of land on which he can grow corn, wheat or soybeans. Each acre of corn costs Rs. 100 for preparation requires 7 man-days of work and yields a profit of Rs. 30. An acre of wheat cost Rs. 120 to prepare, requires 10 man-days of work and yield a profit of Rs. 40. An acre of soybeans cost Rs. 70 to prepare requires 8 man-days of work and yields a profit of Rs. 20. If the farmer has Rs. 1,00,000 for preparation and can count on 8,000 man-

days of work, how many acres should be allocated to each crop to maximize profit ? 20

(b) Define validity of a test and describe various types of validity. 20

(c) What are the assumptions underlying the common queuing models ? Give the role of queuing theory in decision-making and discuss its applications. 20